Proposed solution of problem 1126 IIME Journal vol.14, No.4, 2006, (deadline 10–01–06)

Find a rational function f(x) with integer coefficient such that

$$\cos\theta = f(\sin\theta - \cos\theta)$$

is an identity or prove that no identity of this form exists

Proof No identity of the given form can exist because $\sin \theta - \cos \theta = 1$ for $\theta_1 = \frac{\pi}{2}$ and $\theta_2 = \pi$ but $\cos \pi = -1$ while $\cos \frac{\pi}{2} = 0$.