Proposed problem to Mathematical Reflections

Find the limit
$$\lim_{n\to\infty} n\sin\left(\left((2\pi n)^p+2^p\pi^p n^{p-1}p\right)^{1/p}\right)$$

Answer $\pi(1-p)$

Proof

$$\left((2\pi n)^p+p2^p\pi^p n^{p-1}\right)^{1/p}=2\pi n\left(1+\frac{p2^p\pi^p n^{p-1}}{(2\pi n)^p}\right)^{1/p}=2\pi n\left(1+\frac{p}{n}\right)^{1/p}=$$
$$=2\pi n\left(1+\frac{1}{n}+\frac{1}{2p}\left(\frac{1}{p}-1\right)\frac{p^2}{n^2}+O(\frac{1}{n^3})\right)=2\pi(n+1)+\left(\frac{1}{p}-1\right)\frac{\pi p}{n}+$$
$$+O(\frac{1}{n^2})$$

thus using $\sin x = x+O(x^3)$, $x\to 0$

$$n\sin\left(\left((2\pi n)^p+p2^p\pi^p n^{p-1}\right)^{1/p}\right)=n\sin\left(\left(\frac{1}{p}-1\right)\frac{\pi p}{n}+O(\frac{1}{n^2})\right)=$$
$$=\left(\frac{1}{p}-1\right)\pi p+O(\frac{1}{n})\to\ \pi(1-p)$$

Roma 04/26/2023

Best regards
Paolo Perfetti

Perfetti Paolo, dipartimento di matematica, Università degli studi di Tor Vergata Roma, via della ricerca scientifica, 00133 Roma, Italy – email: perfetti@mat.uniroma2.it