obtain that $\lim_{n\to\infty} A_n = f(0)$ and $\lim_{n\to\infty} \ln G_n = \ln f(0)$. By the squeeze principle $\lim_{n\to\infty} U_n = f(0)$.

Also solved by DIMITRIOS KOUKAKIS, Kato Apostoloi, Greece; PAOLO PERFETTI, Dipartimento di Matematica, Università degli studi di Tor Vergata Roma, Rome, Italy; JOEL SCHLOSBERG, Bayside, NY, USA; and the proposer. Schlosberg used the harmonic mean instead of the geometric mean.

3686. [2011 : 456, 458] *Proposed by Michel Bataille, Rouen, France.*

Let a, b, and c be real numbers such that $abc = 1$. Show that

$$\left(a - \frac{1}{a} + b - \frac{1}{b} + c - \frac{1}{c}\right)^2 \le 2\left(a + \frac{1}{a}\right)\left(b + \frac{1}{b}\right)\left(c + \frac{1}{c}\right).$$

I. Solution by Arkady Alt, San Jose, CA, USA; and Titu Zvonaru, Cománeşti, Romania(independently).

Let $x = a+b+c$ and $y = ab+bc+ca$. Since $abc = 1$, the difference between the two sides of the inequality is

$$\begin{aligned}
&2(a^2+1)(b^2+1)(c^2+1) - (a+b+c-bc-ca-ab)^2 \\
&\quad = 2(2 + a^2b^2 + b^2c^2 + c^2a^2 + a^2 + b^2 + c^2) - (a+b+c-ab-bc-ca)^2 \\
&\quad = 2(2 + y^2 - 2x + x^2 - 2y) - (x-y)^2 = (x^2 + y^2 + 2xy - 4x - 4y + 4) \\
&\quad = (x+y-2)^2 \ge 0.
\end{aligned}$$

Equality occurs if and only if $a+b+c+ab+bc+ca-2 = 0$.

II. Solution by Šefket Arslanagić, University of Sarajevo, Sarajevo, Bosnia and Herzegovina; Oliver Geupel, Brühl, NRW, Germany; Kee-Wai Lau, Hong Kong, China; Salem Malikić, student, Simon Fraser University, Burnaby, BC; and Phil McCartney, Northern Kentucky University, Highland Heights, KY, USA (independently).

Let

$$f(a,b) = 2(a^2+1)(b^2+1)(a^2b^2+1) - (a^2b + ab^2 + 1 - a - b - a^2b^2)^2.$$

Replacing c by $1/ab$, we find that the difference of the two sides of the inequality is

$$\begin{aligned}
a^{-2}b^{-2}f(a,b) &= a^{-2}b^{-2}(1 + a + b - 2ab + a^2b + ab^2 + a^2b^2)^2 \\
&= (c + ac + bc - 2 + a + b + ab)^2 \ge 0,
\end{aligned}$$

from which the result follows, with the foregoing condition for equality.

III. Solution by AN-anduud Problem Solving Group, Ulaanbaatar, Mongolia; and Paolo Perfetti, Dipartimento di Matematica, Università degli studi di Tor Vergata Roma, Rome, Italy(independently).

We can determine nonzero real values u, v, w for which $a = v/w$, $b = w/u$ and $c = u/v$. Then

$$\begin{aligned} a - \frac{1}{a} + b - \frac{1}{b} + c - \frac{1}{c} &= (uvw)^{-1}(uv^2 - uw^2 + vw^2 - vu^2 + wu^2 - wv^2) \\ &= (uvw)^{-1}(u-v)(v-w)(w-u) \end{aligned}$$

and

$$\begin{aligned} &2\left(a + \frac{1}{a}\right)\left(b + \frac{1}{b}\right)\left(c + \frac{1}{c}\right) \\ =&(uvw)^{-2}[(1^2+1^2)(v^2+w^2)][(u^2+v^2)(u^2+w^2)] \\ =&(uvw)^{-2}[(v+w)^2+(v-w)^2][(u^2+vw)^2+u^2(v-w)^2] \\ =&(uvw)^{-2}[((v+w)(u^2+vw)+u(v-w)^2)^2+(u(v+w)(v-w)-(v-w)(u^2+vw))^2] \\ =&(uvw)^{-2}[(uv(u+v)+vw(v+w)+wu(u+v)-2uvw)^2+(v-w)^2(u-v)^2(u-w)^2]. \end{aligned}$$

The desired inequality follows; equality occurs if and only if

$$uv(u+v) + vw(v+w) + wu(u+w) - 2uvw = 0,$$

which is equivalent to $ac + a + ba + b + bc + c - 2 = 0$.

Also solved by DIMITRIOS KOUKAKIS, Kato Apostoloi, Greece; and the proposer. Several solvers pointed out that equality occurs for infinitely many triples. Since the condition can be written as $(a + a^{-1}) + (b + b^{-1}) + (c + c^{-1}) = 2$, it is clear that not all of a, b, c can be positive. McCartney found this in a more precise way by noting that

$$f(a,b) - 16 = (1+a)(1+b)(1+ab)(1+a+b-6ab+a^2b+ab^2+a^2b^2)$$

and observing that the last factor is positive for positive a, b, c since $6ab \le 1+a+b+a^2b+ab^2+a^2b^2$ by the Arithmetic-Geometric Means Inequality. Since $abc = 1$, the condition for equality can be written as

$$a(a+1)b^2 + (a-1)^2b + (a+1) = 0.$$

This is a quadratic equation in b with discriminant $a^4-8a^3-2a^2-8a+1 = (a^2-1)^2-8a(a^2+1)$. Select any negative value of a to get a positive discriminant, solve the quadratic for b and set $c = a^{-1}b^{-1}$ to obtain equality.

3687. [2011 : 456, 458] *Proposed by Albert Stadler, Herrliberg, Switzerland.*

Let n be a nonnegative integer. Prove that

$$\sum_{k=0}^{\infty} \frac{k^n}{k!}\left(k + 1 - \frac{1}{k!}\int_1^{\infty} e^{-t}t^{k+1}dt\right) = \sum_{k=0}^{n} \frac{S(n,k)}{k+2},$$

where k^n is taken to be 1 for $k = n = 0$ and $S(n,k)$ are the Stirling numbers of the second kind that are defined by the recursion

$$S(n,m) = S(n-1,m-1) + mS(n-1,m), S(n,0) = \delta_{0,n}, S(n,n) = 1\,.$$

[*Ed.: The proposer's original problem erroneously had an extra term $k!$ in the denominator that was not caught by the editorial board. As a result, no other*