Workshop

Dynamical Systems: from geometry to mechanics

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Titles and abstracts:

- Marie-Claude Arnaud (Université d'Avignon, France)
  Denjoy dynamics and horseshoes on surfaces.
  I will continue the study of Denjoy subsystem, a notion that we constructed with Patrice Le Calvez in 2017. After introducing Denjoy subsystems and the notion of rotation number that can be attached to them, I will explain in which sense they are parts of horseshoe dynamics:
  - any horseshoe of a surface contains a lot of continuous Denjoy dynamics with various rotation numbers;
  - in the specific case of generic area preserving twist diffeomorphism of the annulus, most of the Aubry-Mather sets are contained in some horseshoes with no conjugate point.

- Viviane Baladi (LPSM, Sorbonne Université, France)
  On the measure of maximal entropy of Sinai billiards (joint with Mark Demers).
  Sinai billiards maps and flows are uniformly hyperbolic, however grazing orbits give rise to singularities. Most existing works on the ergodic properties of billiards are about the SRB measure (i.e., the Liouville measure in the case of flows), for which exponential mixing is known (both in discrete and continuous time). Another natural equilibrium state is the measure of maximal entropy. Since the discrete-time billiard is discontinuous, the mere existence of this measure is not granted a priori. With Mark Demers, we have recently constructed a measure of maximal entropy and shown that it is Bernoulli and has full support. I will also discuss conditions ensuring that the measure of maximal entropy differs from the SRB measure.

- Eric Bedford (Stony Brook University, USA)
  Dynamics of conservative holomorphic maps of surfaces of complex dimension 2.
  We will discuss the dynamics of conservative holomorphic maps with a focus on the possibilities for rotation domains.

- Pierre Berger (Université Paris 13, France)
  Emergence of non-ergodic dynamics.

Recently wild dynamics were shown locally typical of in the sense of Kolmogorov. These dynamics are wild in the sense that they seem difficult to approximate by a finitely ergodic system. In order to quantify this complexity, I introduced the emergence $\mathcal{E}(\epsilon)$ as the minimal number N of probability measures $(\mu_i)_{1\leq i\leq N}$ such that the empirical function $x \mapsto \mathsf{e}_k(x) := \frac{1}{k}\sum_{i=1}^{k} \delta_{f^i(x)}$ satisfies:

$$\limsup_{k\to\infty} \int_M d_{W_1}(\mathsf{e}_k(x), \{\mu_i : \ 1 \leq i \leq N\}) dLeb < \epsilon \ ,$$

where d_{W_1} is the 1-Wasserstein metric on the space of probability measures. I will present a program which aims to:

(1) Show the typicality of dynamics with high emergence.
(2) Describe dynamics with high emergence using a dictionary with the notion of entropy.

Such a program will be illustrated by recent achievements in several collaborations, with Bochi, Biebler, Talebi, Turaev, for symplectic, differentiable or holomorphic dynamics.

- Rafael de la Llave (Georgia Institute of Technology, USA)
Recent progress in the geometric program for Arnol'd diffusion.
The main idea in the geometric program is to find simple geometric structures whose presence implies a rich orbit structure. The existence of the geometric structures in concrete examples should be verifiable in concrete calculations.
By now there are several of these structures that have been identified (undoubtedly, many more remain). We will concentrate in one of those and report on consequences found. They involve applications to concrete problems of interest in astrodynamics.
We will describe how the hypothesis of the mechanism can be obtained by computer assisted proofs. In the perturbative setting, verifying some simple transversality properties, establishes the presence for open sets of parameters. These transversality properties, are very generic and do not require non-generic assumptions such as convexity. We will report on joint work with M. Capinski, Q. Chen, M. Gidea, T. Seara as well as on work of (M.Capinski, M. Gidea).

- Diaz Madrigal (Universidad de Sevilla, Spain)
Slope problems in the theory of semigroups of holomorphic self-maps of the unit disc. Given a semigroup (φ_t) of holomorphic self-maps of the unit disc $\mathbb{D}$ and fixed a point $z \in \mathbb{D}$, the function $t \in [0, +\infty) \mapsto \varphi_t(z) \in \mathbb{D}$ can be seen as the trajectory of a certain vector field. Indeed, many times these trajectories land at concrete points in the circle. From a dynamical point of view, this suggests the question of when this happens with a definite slope. In this talk, we give a panoramic view of this problem paying special attention to several very recent developments which show the important role of Hyperbolic Geometry in this question and also that some kind of wild behaviour is possible.

- Christos Efthymiopolus (Academy of Athens, Greece)
Semi-analytical approaches to Arnold diffusion.
The use of computer-assisted high order normal form calculations allows one to probe Arnold diffusion in generic a priori stable nearly-integrable Hamiltonian systems with three or more degrees of freedom. The talk will review some progress in this domain in the last years, focusing on: i) probes of the relation between the speed of Arnold diffusion and the size of the remainder of the optimal Nekhoroshev normal form in simply and doubly resonant domains (Efthymiopoulos 2008; Efthymiopoulos and Harsoula 2013; Cincotta et al. 2014), and ii) semi-analytical calculation of the adiabatic evolution of the action variables along simple resonances based on a (quasi-) stationary phase approximation (collaboration with M. Guzzo and R. Paez, Guzzo et al. 2018).
References:

  - Efthymiopoulos C., On the Connection between the Nekhoroshev theorem and Arnold diffusion. Cel. Mech. Dyn. Astron. 102, pp.49-68 (2008).
  - Efthymiopoulos, C. and Harsoula, M., The speed of Arnold Diffusion. Physica D 251, pp. 19-38, 2013.
  - Cincotta, P., Efthymiopoulos, C., Giordano, C.M., and Mestre, M., Chirikov and Nekhoroshev diffusion estimates: bridging the two sides of the river. Physica D, 266, pp.49-64, 2014.
  - Guzzo, M., Efthymiopoulos, C., Paez, R., Semi-analytic computations of the speed of Arnold diffusion along single resonances in a priori stable Hamiltonian systems. Submitted, eprint arXiv:1812.05430, 2018.

- Nuria Fagella (Universitat de Barcelona, Spain)
Wandering domains from the inside. In the setup of transcendental entire functions of the complex plane, we make first steps towards a classification of wandering domains, in terms of the behavior of the orbits inside these Fatou components.

- Albert Fathi (Georgia Institute of Technology, USA)
Singularities of solutions of the Hamilton-Jacobi equation. A toy model: distance to a closed subset.
This is a joint work with Piermarco Cannarsa and Wei Cheng. If A is a closed subset of the Euclidean space $\mathbb{R}^k$, the Euclidean distance function $d_A : \mathbb{R}^k \longrightarrow [0, +\infty[$ is defined by

$$d_A(x) := \min_{a \in A} \|x - a\|.$$

This function is Lipschitz, therefore differentiable almost everywhere. We will give topological properties of the set $\mathrm{Sing}(F)$ of points in $\mathbb{R}^k \setminus M$ where F is not differentiable. For example it is locally connected. We will also discuss the homotopy type of $\mathrm{Sing}(F)$. Although, we will concentrate on d_A, we will explain that it is a particular case of a more general result on the singularities of a viscosity solution $F : \mathrm{Sing}(F) \times]0, +\infty[\longrightarrow \mathbb{R}$ of the evolution Hamilton-Jacobi equation

$$\partial_t F + H(x, \partial_x F) = 0,$$

where $H : \mathbb{R}^k \times \mathbb{R}^k \longrightarrow \mathbb{R}^k$, $(x, p) \longmapsto H(x, p)$ is a C^2 Tonelli Hamiltonian, i.e. convex and superlinear in the momentum p. If time permits we will explain the methods of proof for the case of d_A.

- John Erik Fornaess (Norwegian University of Science and Technology, Norway)
  Henon mappings. I will talk about recent joint work with Leandro Arosio, Anna Benini and Han Peters. Complex dynamics of entire functions in one complex dimension has been thoroughly investigated. The dynamics of complex polynomial Henon mappings has also been much studied by Bedford, Smillie and others. Our work is concerned with mixing the two theories in order to understand entire Henon mappings.

- Vadim Kaloshin (University of Maryland, USA)
  Can you hear the shape of a drum and deformational spectral rigidity of planar domains.
  M. Kac popularized the question 'Can you hear the shape of a drum?'. Mathematically, consider a bounded planar domain Ω and the associated Dirichlet problem $\Delta u + \lambda^2 y = 0,\ u|\partial\Omega = 0$. The set of λ's such that this equation has a solution, denoted $L(\Omega)$, is called the Laplace spectrum of Ω. Does Laplace spectrum determine Ω? In general, the answer is negative. Consider the billiard problem inside Ω. Call the length spectrum the closure of the set of perimeters of all periodic orbits of the billiard. Due to deep properties of the wave trace function, generically, the Laplace spectrum determines the length spectrum. We show that any generic axis symmetric planar domain with sufficiently smooth boundary and sufficiently close to a circle, is dynamically spectrally rigid, i.e., can't be deformed without changing the length spectrum. This partially answers a question of P. Sarnak. This is a joint work with J. De Simoi and Q. Wei.

- Stefano Luzzatto (Abdus Salam International Centre for Theoretical Physics (ICTP), Italy)
  Recent progress on the Viana conjecture.
  I will discuss some questions related to the Viana conjecture on the existence of SRB measures for systems with non-zero Lyapunov exponents. In particular I will state a new result on the existence of SRB measures for surface diffeomorphisms under quite weak non-uniform hyperbolicity conditions. The proof involves a new technique for the construction of Young Towers and a key technical point is a new general result on the hyperbolic properties of pseudo-orbits in non-uniformly hyperbolic sets.

- Stefano Marmi (Scuola Normale Superiore di Pisa, Italy)
  Birkhoff sums of diophantine interval exchange maps.
  Interval exchange transformations (i.e.t.) generalize circle maps. They share some of the rigidity properties of circle maps (in particular smooth perturbations of linear i.e.t. with diophantine rotation number are smoothly conjugated to the linear i.e.t. with the same rotation number) but they also exhibit a number of new phenomena (they need not be uniquely ergodic, typical non-toral i.e.t.'s are weakly mixing, etc). We introduce a Diophantine

condition on rotation numbers of interval exchange maps (i.e.m) and translation surfaces: the dual Roth type condition is a condition on the backward rotation number of a translation surface. Under the dual Roth type condition, we associate to a class of functions with subpolynomial deviations of ergodic averages (corresponding to relative homology classes) distributional limit shapes. Joint work with Corinna Ulcigrai and Jean-Christophe Yoccoz.

- Gabriella Pinzari (Università degli Studi di Padova, Italy)
  Existence of a region with stable and whiskered tori in the retrograde three-body problem.
  The three body problem in the planetary case allows, in principle, infinite systems of coordinates (obtained acting on the degenerate degrees of freedom) that allow to reduce invariance by rotation. In my talk I shall show that there exist two choices of coordinate systems that allow to prove, in the retrograde problem: maximal tori on one side; whiskered tori on the other side, and, finally, a region where such two families of tori co–exist.
  The talk is based on [G. Pinzari, JMP 2018] and work in progress by the author.

- Jasmin Raissy (Université de Toulouse, France)
  A dynamical Runge embedding of $\mathbb{C}\times\mathbb{C}^*$ in $\mathbb{C}^2$.
  I will present the construction of a family of automorphisms of $\mathbb{C}^2$ having an invariant, non-recurrent Fatou component biholomorphic to $\mathbb{C}\times\mathbb{C}^*$ and which is attracting, in the sense that all the orbits converge to a fixed point on the boundary of the component. Such component is obtained by globalizing, using a result of Forstneric, a local construction, which allows to create a global basin of attraction for an automorphism, and a Fatou coordinate on it. Such Fatou coordinate is a fiber bundle map on $\mathbb{C}$, whose fiber is $\mathbb{C}^*$, forcing the global basin to be biholomorphic to $\mathbb{C}\times\mathbb{C}^*$. The most subtle point is to show that such a basin is indeed a Fatou component. This is done exploiting Pöschel's results about existence of local Siegel discs and suitable estimates for the Kobayashi distance. This construction gives an example of a Runge embedding of $\mathbb{C}\times\mathbb{C}^*$ in $\mathbb{C}^2$, since attracting Fatou components are Runge. (Joint work with Filippo Bracci and Berit Stensønes).

- David Sauzin (CNRS-IMCCE, France)
  KAM tori are no more than sticky (Joint work with Bassam Fayad, CNRS IMJ-PRG, Paris).
  When a Gevrey smooth perturbation is applied to a quasi-convex integrable Hamiltonian, it is known that the KAM invariant tori that survive are "sticky", i.e., doubly exponentially stable. We show by examples the optimality of this effective stability.

- Tere M-Seara (Universitat Politécnica de Catalunya, Spain)
  A general mechanism for instability in Hamiltonian systems.
  The problem of Arnold diffusion consists in studying in which Hamiltonian systems the effects of perturbations can accumulate over time to produce effects much larger than the size of the perturbations. Specially in integrable

systems. We will describe a recent mechanism based on the presence of Normally hyperbolic manifolds. The mechanism is rather robust. It does not need that the perturbations are Hamiltonian (applies to small dissipation problems or for space craft maneuvers that involve burns), can be applied to to concrete problems and enjoys remarkable genericity properties since it does not require non-generic assumptions such as convexity. Joint work with M. Gidea and R. de la Llave.

- Amie Wilkinson (University of Chicago, USA)
  Pathology and asymmetry.
  I will report on joint work with Daniela Damjanović and Disheng Xu. We explore a type of centralizer rigidity for certain partially hyperbolic diffeomorphisms: if the centralizer has high enough rank, then it contains a Lie group. The hidden player in these results is the notion of pathological foliations introduced by Mike Shub and myself in 1998.