Explicit Characterization of Consensus in a Distributed Estimation Problem on Chain Graphs

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Abstract

This Abstract briefly describes the original results that are to be submitted to the international journal *IEEE Transactions on Automatic Control*. In particular, the problem of looking for time-varying vectors $\hat{\Theta}(t)$ – named Θ-estimates – which exponentially converge to the unknown constant parameter vector $\Theta \in \mathbb{R}^m$ defined by the set of linear time-varying equations:

$$y_i(t) \quad = \quad \phi_i^{\mathrm{T}}(t)\Theta, \quad i = 1, \ldots, p \tag{1}$$

is addressed, where y_i are the locally measured outputs and $\phi_i(\cdot) : \mathbb{R}_0^+ \to \mathbb{R}^m$ are the local regressor vectors, $i = 1, \ldots, p$, each of them assumed to be available at the running time at each node of the graph. This general problem is referred to as the estimation problem for (1) on a graph. Now, when there are no edges connecting nodes of the graph that are far from each other, the problem passes from being full-graph-knowledge based to being partial-graph-knowledge based. This way, the burden of information that has to be communicated to the various measurement/estimation nodes of the graph might be (even largely) reduced. Specifically, the original contribution consists in showing that, under the weakest Θ-identification condition on the graph [1]:

(*Cooperative PE Condition*) The corresponding regressor matrix $\Phi^{\mathrm{T}}(\cdot) \in \mathbb{R}^{p \times m}$

$$\Phi^{\mathrm{T}}(\cdot) \quad = \quad \begin{bmatrix} \phi_1^{\mathrm{T}}(\cdot) \\ \cdots \\ \phi_p^{\mathrm{T}}(\cdot) \end{bmatrix}$$

is persistently exciting (PE), i.e., there exist (known) positive reals c_p and T_p such that the following condition [$\mathbb{I} \in \mathbb{R}^{m \times m}$] holds:

$$\int_t^{t+T_p} \Phi(\tau)\Phi^{\mathrm{T}}(\tau)\mathrm{d}\tau \quad \geqq \quad c_p\mathbb{I}, \quad \forall\, t \geq 0, \tag{2}$$

a set of suitably tailored differential equations for the time-dependent vectors $\hat{\Theta}^{[i]}(t)$, all of them converging to the unknown Θ and by thus achieving *consensus*, can be (redundantly) designed at each node $i = 1, \ldots, p$. This is proved under the condition that nodes undirectedly connected in series (undirected chain graph [2]) are considered, so that each estimation scheme at the node can share information – namely, its own Θ-estimate – with the neighbours only, one for node 1 and p, two for the remaining nodes. Thus the aforementioned problem belongs to the class of problems known as *multi-agent estimation with consensus*.

The general scenario is the one given by a multisensor network in which a distributed parameter estimator is to be designed on the basis of distributed sensing. This situation typically arises in the case in which mobile sensors measure the distribution of an unknown quantity over a field and, due to the size of the region, it is computationally inefficient or just infeasible to visit every point in the space to collect data ([3]). Now, agents at the nodes face a local identification problem, in which they cannot consistently estimate the parameter vector in isolation, so they have to engage in communication with their neighbours. In particular, estimate-consensus has to be achieved through a sort of penalization of the mismatch between the parameter estimates. Indeed, in this Abstract, by assessing the positive definite nature of a quadratic form associated with a tridiagonal block structure, we prove the exponential achievement – with an explicit characterization of the exponential convergence – of the distributed parameter estimation task on an undirected chain graph, in which an original neighbourhood-based decentralized parallel architecture (aiming at reducing the computational burden of a centralized estimation scheme on the graph) is adopted. It is not necessary for Θ to be fully identifiable at each node. Instead, it is sufficient for the nodes in the graph to cooperatively provide information at the local stage. Indeed, the derivations here are essentially in accordance with [1], however, in contrast to [1], which – for general graph topologies – uses weaker contradiction arguments to prove exponential consensus, here original proofs of convergence are able to provide an explicit characterization of the exponentially achieved consensus.

Nevertheless, the problem of identifying time-varying parameters that are periodic with known periods can be innovatively solved as well. This constitutes an additional original contribution of the paper: adaptive tools can be directly replaced by repetitive learning ones [4] within the same theoretical framework, where the asymptotic consensus is successfully ensured under identification mechanisms based on the information exchange between neighbours.

References

[1] W. Chen, C. Wen, S. Hua, and C. Sun, Distributed cooperative adaptive identification and control for a group of continuous-time systems with a cooperative PE condition via consensus, *IEEE Transactions on Automatic Control*, vol. 59(1), pp. 91-106, 2014.

[2] A. Concas, L. Reichel, G. Rodriguez, and Y. Zhang, Chained graphs and some applications, *Applied Network Science*, vol. 6(39), 2021.

[3] A.S. Matveev, M. Almodarresi, R. Ortega, A. Pyrkin, and S. Xie, Diffusion-based distributed parameter estimation through directed graphs with switching topology: Application of dynamic regressor extension and mixing, *IEEE Transactions on Automatic Control*, vol. 67(8), pp. 4256-4263, 2022.

[4] C.M. Verrelli, Exponential output tracking via a saturated RLC for a class of nonlinear systems under PSD of periodic references, *Systems and Control Letters*, vol. 192, 2024.