$$(*) \qquad e^{z+w} = e^z e^w, \quad z, w \in \mathbf{C}.$$

Here is a proof of (*) in which we leave out the nontrivial justification of some of the steps. We have

$$\begin{aligned}
e^{z+w} &= \sum_{n=0}^{\infty} \frac{(z+w)^n}{n!} \\
&= \sum_{n=0}^{\infty} \left(\sum_{m=0}^{n} \frac{z^m}{m!} \frac{w^{n-m}}{(n-m)!} \right) \\
&= \sum_{m=0}^{\infty} \left(\sum_{n=m}^{\infty} \frac{z^m}{m!} \frac{w^{n-m}}{(n-m)!} \right) \\
&= \sum_{m=0}^{\infty} \left(\frac{z^m}{m!} \sum_{n=m}^{\infty} \frac{w^{n-m}}{(n-m)!} \right) \\
&= \sum_{m=0}^{\infty} \left(\frac{z^m}{m!} \sum_{n=0}^{\infty} \frac{w^n}{n!} \right) \\
&= \sum_{m=0}^{\infty} \frac{z^m}{m!} e^w \\
&= e^z e^w.
\end{aligned}$$